GUIDANCE OF SPACE VEHICLES BY RADIO MEASUREMENTS AND COMMAND

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1. INTRODUCTION

THE guidance employed during the early powered flight of a space vehicle is referred to as injection, boost or ascent guidance. The techniques are almost the same as those used in the guidance of ballistic missiles and they are not described in this paper. Instead, the correction of space trajectories by small impulse-type maneuvers is discussed, since injection guidance alone would not usually be sufficiently accurate for advanced space missions.

Such so-called midcourse maneuvers are applied by means of a small rocket motor mounted in the spacecraft. The magnitude and direction of the correcting impulse is computed on the ground from radio measurements and, prior to the maneuver, the appropriate commands are sent by radio to the probe.* In addition to presenting the theoretical foundations, this paper gives representative figures for the errors both in determining the orbit and in applying the correction. Reference is also made to tracking sites and the mechanization in the spacecraft.

2. THEORY OF MIDCOURSE MANEUVERS

At this point it is convenient to assume that the actual trajectory followed by a vehicle differs only slightly from some precalculated standard trajectory. Linear perturbation theory may then be applied to all calculations dealing with coordinate variations and small velocity increments (for correcting the trajectory). Although for most purposes the approximations of linear perturbations are good, the theory is invoked more as a convenience than as a necessary step in the calculations. The theory is used to carry out first-order analyses but, where necessary, iterative procedures would refine the approximations.

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* The terms space vehicle, spacecraft and space probe are used synonymously.

If a probe reaches the desired destination point at a given time t_2 on the ideal or standard trajectory, then, because of injection errors, the probe will not, in general, reach the same point at time t_2 on an actual trajectory unless some correction is applied. Let the differences in the coordinates of position on the standard and the actual trajectory (in the absence of a correction) be $(\delta x, \delta y, \delta z)$ at time t_2. It is shown in Appendix A that, in order to correct the trajectory by applying a velocity-impulse with components (v_x, v_y, v_z) at some previous time t_1,

$$\begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}_{t_2} = -H \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_{t_1} \tag{1}$$

where H is a (3×3) matrix, the elements of which can be determined from computations on the standard trajectory for any given t_1 and t_2. Components $(\delta_x, \delta_y, \delta_z)$ at time t_2 can be computed indirectly from measured data and so the three velocity components necessary for the correction of the trajectory are determined by Eq. (1). This would, however, guide the space probe to intercept a given moving point in space at a given time. The latter restriction would usually be unnecessary, small variations in the flight time would be permissible in most cases.

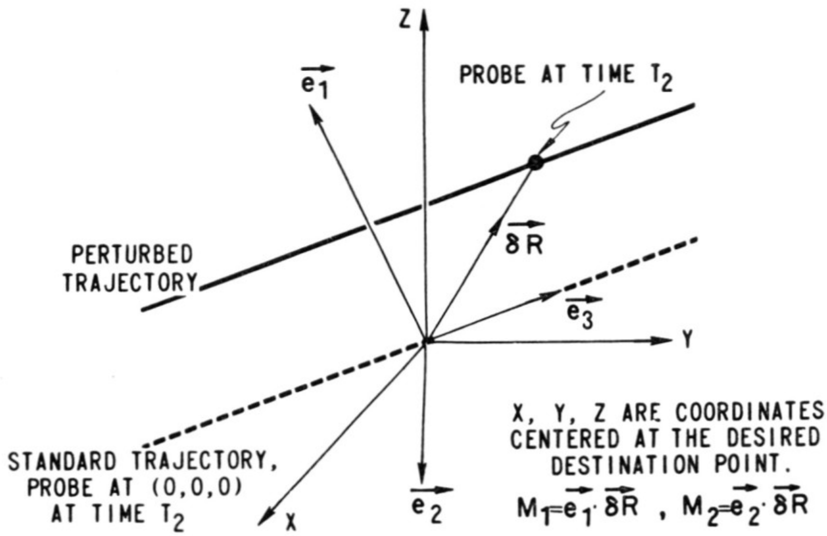

FIG. 1. Definition of miss-components M_1 and M_2.

In order to allow variations of flight time, a new set of rectangular axes is taken (Fig. 1) centered at the probe position at time t_2 on the standard trajectory. The new axes are defined by the unit base vectors $\vec{e}_1$, $\vec{e}_2$, and $\vec{e}_3$, where $\vec{e}_3$ is along the probe velocity vector at time t_2 on the standard trajectory. Since $\vec{e}_2$ is in the xy-plane and perpendicular to $\vec{e}_3$ and $\vec{e}_1$, it completes the set of mutually perpendicular base vectors. If then $\overrightarrow{\delta R}$ is the vector displacement corresponding to $(\delta x, \delta y, \delta z)$ at time t_2, three new miss-components may be defined

$$M_1 = \vec{e}_1 \cdot \overrightarrow{\delta R}, \quad M_2 = \vec{e}_2 \cdot \overrightarrow{\delta R}, \quad M_3 = \vec{e}_3 \cdot \overrightarrow{\delta R} \tag{2}$$

Assuming that, as a first-order approximation,* perturbed trajectories have the same velocity direction at time t_2, then M_1 and M_2 are the miss-components at some time different to t_2, and M_3 is associated only with time of flight variations. Thus

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} l_2 \, m_2 \, n_2 \\ l_1 \, m_1 \, n_1 \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}_{t_2} \tag{3}$$

where (l_1, m_1, n_1) and (l_2, m_2, n_2) are the direction cosines of $\vec{e}_1$ and $\vec{e}_2$, respectively. Indicating the (2×3) matrix of Eq. (3) by N, Eq. (1) can be modified to

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = -NH \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_{t_1} \tag{4}$$

If the velocity components satisfy the above two equations, the trajectory would be corrected to pass through the desired end-point at some unspecified time. However, the three components have to satisfy only two equations, therefore there is one degree of redundancy, which may be used for one of the following:

(a) to minimize the magnitude of the correction
(b) to apply a geometrical constraint to the maneuver for the sake of practical convenience.
(c) to control an additional destination variable such as speed or time of arrival.

In the first case let

$$NH = [K_{ij}] \tag{5}$$

It can be shown that the magnitude of (v_x, v_y, v_z) is a minimum when

$$\begin{vmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ v_x & v_y & v_z \end{vmatrix} = 0 \tag{6}$$

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* This approximation may be improved by taking out the effect of the attraction of the destination planet. Such a procedure is convenient for first-order error analyses.

Equation (6) defines a plane in which the correcting velocity vector would always be, when applied at a given time, t_1. The plane is independent of injection errors. The plane may be referred to as the most efficient or critical plane, the normal to that plane being the noncritical direction. The latter depends on the trajectory and the maneuver-time along the trajectory. The non-critical direction is not always approximately parallel to the probe velocity vector.

In practice it may not be convenient to apply the correction in the critical plane. Referring to option (b) above, the correction might be restricted to a plane perpendicular to the probe–sun axis. With a rocket mounted at rightangles to that axis, one face of the spacecraft (carrying solar panels) need not then be turned away from the sun during the mid-course maneuver.

When the rocket thrust vector is restricted to a plane, critical or otherwise, the velocity components would satisfy an equation of the form

$$av_x+bv_y+cv_z = 0 \tag{7}$$

in addition to Eq. (4). To calculate the velocity components, a (3×3) matrix is formed from the (2×3) matrix NH and a, b, c of Eq. (7),

$$P = \begin{bmatrix} NH \\ \hline a \;\; b \;\; c \end{bmatrix} \tag{8}$$

P being a (3×3) matrix. Then

$$\begin{bmatrix} M_1 \\ M_2 \\ 0 \end{bmatrix} = -P \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{9}$$

and

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = -P^{-1} \begin{bmatrix} M_1 \\ M_2 \\ 0 \end{bmatrix} \tag{10}$$

which represents three equations specifying the velocity components (subject to a constraint) in terms of the miss-components M_1 and M_2. The latter would be obtained from the computed orbit according to the measured data.

Actually, in computing the necessary maneuver during a flight, higher order terms would be taken into account. Equation (10) would be employed for each iteration in conjunction with exact integration of the equations of motion in a ground computer.

As regards the amount of maneuvering capability that a spacecraft must carry, this hinges on the accuracy of the injection guidance system, i.e. how accurately the coordinates are controlled at burn-out of the last stage. The calculation of the magnitude of the maneuver in terms of the statistical moment-matrix of injection errors is given in Appendix B.

3. ORBIT DETERMINATION FROM RADIO MEASUREMENTS

In order to compute the required midcourse maneuver the trajectory or orbit of the space probe must be determined from Earth-based measurements. Such measurements would be from radar tracking and possibly photographic detection. Although the latter can be very accurate the processing time is inconviently long, apart from the uncertainties due to weather. Radar tracking is therefore employed to provide the basic data for orbit determination.

Early spacecraft (e.g. the U.S. Pioneer IV lunar probe) have carried only a radio beacon for tracking purposes, i.e. a one-way link. In such cases only angular measurements are possible. Any doppler measurements are dependent on the stability of the probe transmitter frequency and the velocity data becomes virtually useless. Later probes will carry a phase-coherent receiver and transmitter so that the probe transmitter frequency differs from the ground transmitter frequency only by the doppler shift.* Accurate range-rate measurement is then possible. Range data can be obtained by a simple modulation of the carrier signal, but unfortunately such simple schemes require wide bandwidth and consequently high power levels. More advanced modulation techniques are being developed for spacecraft in which long intervals of the carrier signal are modulated in a random fashion; for a given range accuracy the required bandwidth is then much less.

Tracking of space vehicles will be carried out at several sites in different countries but, in the U.S. space program, heavy reliance will be placed on three sites which are being set up especially for the tracking of lunar and interplanetary probes. These are located at Goldstone in the Southern California desert, at Woomera (Australia) and possibly in the southern area of Africa. Each site will eventually have one 85-ft diameter antenna for receiving and one for transmitting. Each site will therefore be able to send commands to the vehicle, apart from measuring the angular position, range-rate and ultimately the range of the space probe. In addition there will be communication links with the main computing center in the United States, in order that the data can be processed and used for the in-flight orbit determination.

The central computer receives therefore many different kinds of measurements from different tracking sites. These measurements are conta-

* The U.S. Pioneer V space probe has such a system.

minated by different kinds of noise. For example angular measurements are corrupted by refraction variations, antenna servo-jitter, slow mechanical deformations of the antenna, etc. Consequently the multitude of data points must be treated statistically (with the appropriate weighting) to determine the orbit which best fits the noisy data. The theoretical procedure is outlined in Appendix C. Apart from describing the method a result is also deduced for estimating the uncertainty in the orbit determination in terms of the expected noise on the measured data. The noise-moment matrix of the uncertainties in the estimates of the six injection coordinates is (Eq. C-12)

$$\overline{\delta V_u \delta V_u^T} = J^{-1} \tag{11}$$

where the general term j_{ij} of the (6×6) J matrix is

$$j_{ij} = \sum_q \sum_{m_q} \frac{1}{\sigma_R^2} \frac{\partial R}{\partial V_i} \frac{\partial R}{\partial V_j} + \frac{1}{\sigma_{\dot{R}}^2} \frac{\partial \dot{R}}{\partial V_i} \frac{\partial \dot{R}}{\partial V_j} + \frac{1}{\sigma_\theta^2} \frac{\partial \theta}{\partial V_i} \frac{\partial \theta}{\partial V_j} + \frac{1}{\sigma_\phi^2} \frac{\partial \phi}{\partial V_i} \frac{\partial \phi}{\partial V_j} \tag{12}$$

for observations of range R, range-rate $\dot{R}$, hour angle θ and declination ϕ from radar sites. M data points are taken from the q'th site $V_i (i = 1, 2, ..., 6)$ are the six injection coordinates and σ^2 denotes the variance of noise on a particular kind of data (modified in the case of significant self-correlation).

Equations 11 and 12 are extremely useful for (a) predicting the precision of orbit determination for given radar sites, (b) specifying the

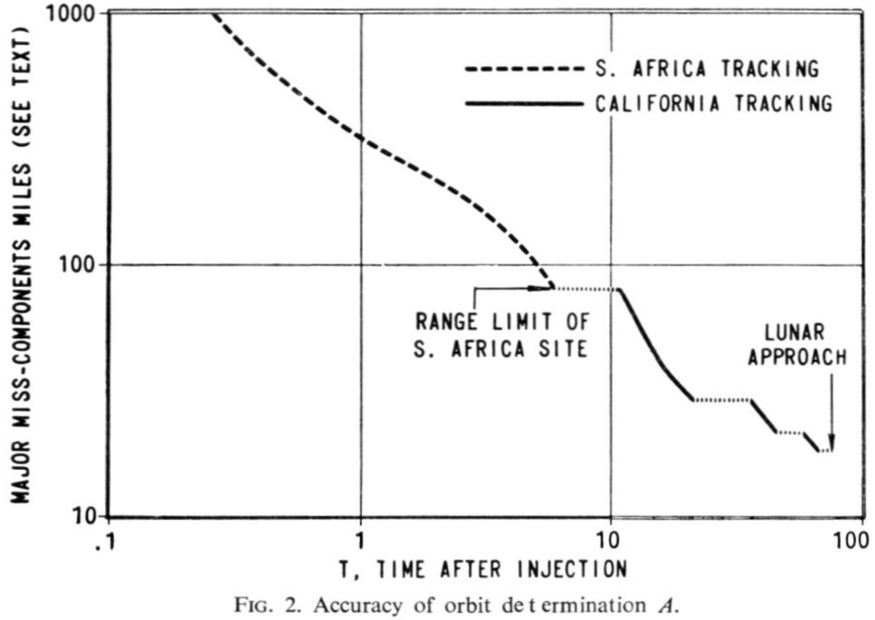

FIG. 2. Accuracy of orbit determination A.

required accuracy of radar tracking to determine orbits to a given accuracy and (c) evaluating the relative usefulness of different kinds of tracking. The results of some representative calculations are shown graphically in Fig. 2 which refers to a 76-hour lunar trajectory injected over the South Atlantic Ocean.

A transformation has been applied to the moment matrix (Eq. 11) to express the results in terms of miss-components at the moon. Thus, if W is the (2×6) matrix relating injection errors to miss components, the dispersion of the miss components M_1 and M_2 is given as

$$\begin{bmatrix} \overline{M_1^2} & \overline{M_1 M_2} \\ \overline{M_2 M_1} & \overline{M_2^2} \end{bmatrix} = WJ^{-1}W^T \tag{13}$$

Contours of constant probability in the $(M_1\, M_2)$-plane are ellipses (assuming Gaussian noise on the measured data) and in Fig. 2 is plotted the semi-major axis of the ellipse which contains 40 per cent of all cases

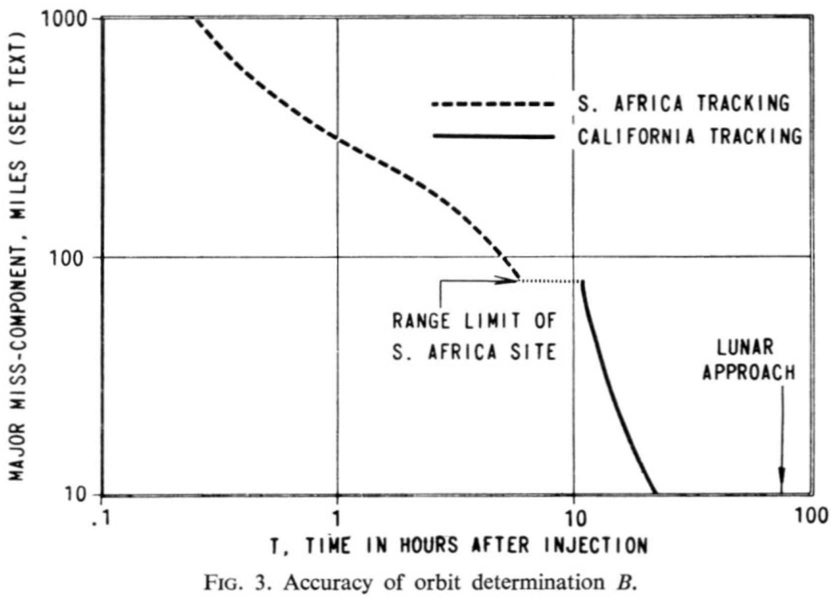

FIG. 3. Accuracy of orbit determination *B*.

(loosely referred to as the major miss-component). If a perfect midcourse maneuver were applied at any time T after injection, Fig. 2 shows the miss that would result at the moon due to the uncertainties in the orbit determination.

Doppler measurements accurate to 1 m/sec were assumed and such data is known to have an influence on the determination more powerful than the angular data. Figure 3 is presented to demonstrate the value of range data from the tracking site at Goldstone, California.

One important conclusion from such studies is that the early tracking data has the strongest effect on the precision of the orbit calculation; if no radar tracking is available until about 5 hours after injection then there is great difficulty in the numerical procedures and the orbit is known only approximately.

4. MECHANIZATION AND SYSTEM PERFORMANCE

It was shown in section 2 that a system of midcourse guidance includes a small rocket with a variable total impulse and the ability to point the

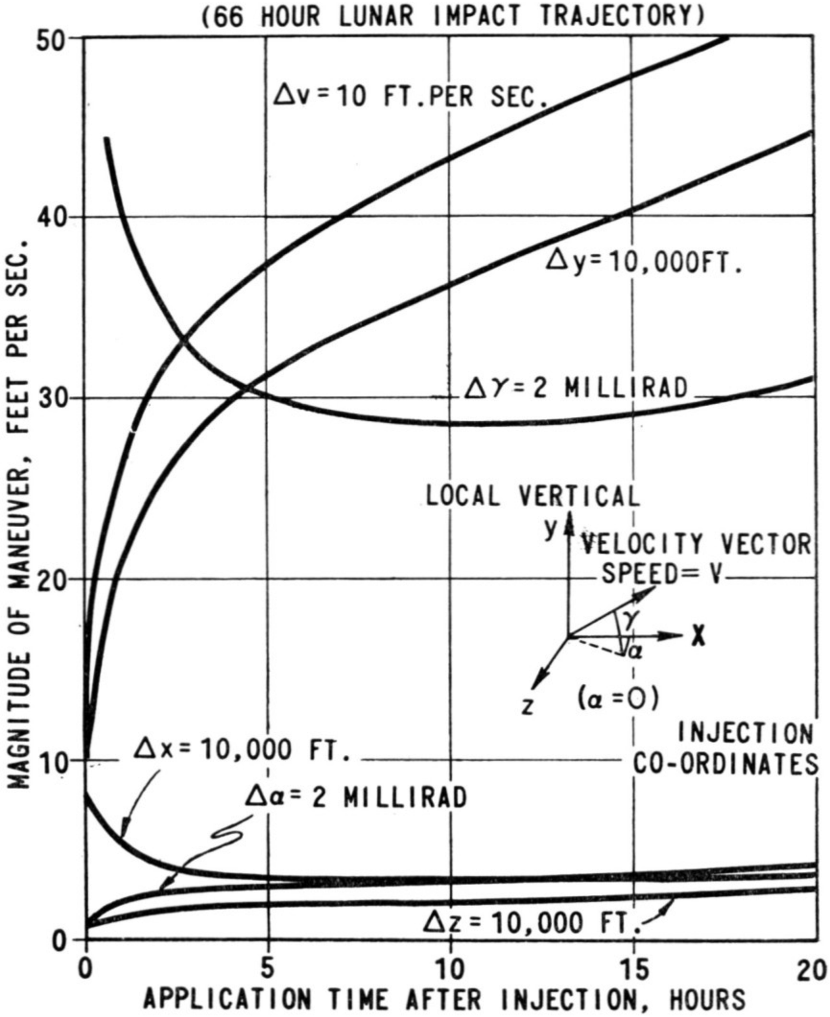

FIG. 4. Midcourse correction of individual injection errors.

thrust vector of the rocket in a desired direction.* Assuming that the rocket is mounted rigidly in the spacecraft, the angular orientation of the latter must be controlled and changed on command from the ground. Attitude control for lunar missions will be with a system of gas jets, but for longer interplanetary missions the addition of flywheel control will be desirable. Provided the electrical power comes from solar panels the weight of a flywheel system does not increase with journey-time.

TABLE 1

Representative figures for the accuracy of midcourse guidance (R.M.S: quotations of errors)

Destination	Orbit determination miss, miles	Rocket motor shut-off			Rocket pointing (100 ft/sec maneuver)			Total Miss, miles
		Coefficient miles per ft/sec.	Assumed error, ft/sec.	Miss, miles	Coefficient miles per degree	Assumed error, degrees	Miss, miles	
Moon	40	22	1	22	38	0.5	19	49
Mars	4000†	3600	1	3600	6200	0.5	3100	6200
Venus	3000†	2500	1	2500	4400	0.5	2200	4500

†) It is assumed that the uncertainty in the Astronomical Unit will be reduced in the near future by more than one order of magnitude (Ref.[2]). Otherwise the miss at Mars and Venus would be the order of 15,000 miles (foı 1 in 2000).

The ease of shutting off and restarting liquid propellant rocket motors makes them attractive for midcourse maneuvers. Thrust levels can be quite low (e.g. 50 lb) and the lower specific impulses of monopropellants are acceptable in simplifying the propulsion unit. A separate tighter form of altitude stabilization would however be required during burning of the rocket. Shut-off of the motor would be dependent upon the integrated output of an accelerometer, mounted with the sensitive axis parallel to that of the thrust vector.

The choice of T, the time of application of the maneuver is influenced by (a) the magnitude of the correction as a function of T, (b) the accuracy of the orbit determination as a function of T and (c) visibility from tracking sites which can send the radio-commands. The calculation of the magnitude of the maneuver is given in Appendix B but the result, as a function of T, is dependent on the relative values and cross correlations of the injection errors. This is illustrated in Fig. 4 where the midcourse maneuver

* This is not, of course, the only possible mechanization. Another method is to make two maneuvers in an invariant direction, although the first maneuver must be made very early to avoid using excessive amounts of rocket propellant. The orbit is not usually determined sufficiently only 2 hours after injection.

to correct one-at-a-time injection errors is plotted against T. It will be observed that they are not all monotonically increasing functions of T.

Taking into account considerations (a), (b) and (c) above, maneuvers in practice would typically be applied 10 to 20 hours after injection on lunar missions and in the first few days of interplanetary journeys.

The accuracy of midcourse guidance depends on (1) the orbit determination, (2) shut-off of the rocket and (3) pointing of the rocket thrust vector. Typical figures for these are summarized in Table 1.

5. CONCLUSIONS

Radio-command midcourse guidance is regarded as having great potential for future lunar and interplanetary missions. It is suitable for ensuring impact on a small preselected area of the surface of the Moon, for guidance prior to the creation of a lunar satellite and for sending a recoverable space probe round the Moon and back to Earth.

Furthermore, provided the measure of the Astronomical Unit is improved, such guidance will ensure approaches of 10,000 to 20,000 miles of the planets Mars and Venus.

6. ACKNOWLEDGEMENTS

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APPENDIX A

CORRECTING VELOCITY COMPONENTS

The coordinates (x_2, y_2, z_2) on a ballistic trajectory at any time t_2 are functions of the coordinates $(x_1, y_1, z_1, \dot{x}_1, \dot{y}_1, \dot{z}_1)$ at any previous time t_1. Hence, by taking only the first order terms of a generalized Taylor expansion,

$$\delta x_2 = \frac{\partial x_2}{\partial x_1}\,\delta x_1 + \frac{\partial x_2}{\partial y_1}\,\delta y_1 + \frac{\partial x_2}{\partial x_1}\,\delta z_1 + \frac{\partial x_2}{\partial x_1}\,\delta \dot{x}_1 + \frac{\partial x_2}{\partial x_1}\,\delta \dot{y}_1 + \frac{\partial x_2}{\partial x_1}\,\partial \dot{z}_1 \quad \text{(A-1)}$$

and similarly for δy_2 and δz_2, where the perturbations are to be interpreted as coordinate variations from a standard trajectory. If, in addition to the six coordinate perturbations at time t_1, a further perturbation is added, an impulse of velocity with components (v_x, v_y, x_z) such that

$$\delta x_2 = -\frac{\partial x_2}{\partial \dot{x}_1} v_x - \frac{\partial x_2}{\partial \dot{y}_1} v_y - \frac{\partial x_2}{\partial \dot{z}_1} v_z \qquad \text{(A-2)}$$

and similarly for δy_2 and δz_2. Then the net result will be that $\delta x_2 = \delta y_2 = \partial z_2 = 0$, since the applied velocity perturbation will exactly cancel the effects of the six coordinate variations at time t_1. Equation (1) is a statement of this result in matrix notation.

Appendix B

MAGNITUDE OF THE MANEUVER

Let δX be a (6×1) matrix of the six injection errors, then the resulting miss components are given by the matrix equation

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = W\delta X \qquad \text{(B-1)}$$

where the (2×6) matrix W is a function of the choice of trajectory. The same miss components can also be achieved by applying a midcourse maneuver with velocity components u_1 and u_2 in a given plane. Thus

$$\begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = K \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad \text{(B-2)}$$

where the (2×2) K matrix is obtainable from the P matrix of equation (10). If u_1 and u_2 are to correct the injection deviations δX,

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -K^{-1}W\delta X \qquad \text{(B-3)}$$

and by multiplying each side of equation B-3 by its transpose

$$\begin{bmatrix} u_1^2 & u_1 u_2 \\ u_2 u_1 & u_2^2 \end{bmatrix} = K^{-1}W\delta X\delta X^T W^{T0} K^{-1T} \qquad \text{(B-4)}$$

Let the six injection errors δX be now considered as random variables; the ensemble of B-4 is

$$\begin{bmatrix} \overline{u_1^2} & \overline{u_1 u_2} \\ \overline{u_2 u_1} & \overline{u_2^2} \end{bmatrix} = K^{-1}W(\overline{\delta X\delta X^T})\, W^T K^{-1T} \qquad \text{(B-5)}$$

But $(\overline{\delta X\, \delta X^T}) = \Lambda$ is in fact the moment matrix of injection errors; it gives the variances and covariances of those errors and would be calculated from the form of the injection guidance system. Therefore

$$\begin{bmatrix} \overline{u_1^2} & \overline{u_1 u_2} \\ \overline{u_2 u_1} & \overline{u_2^2} \end{bmatrix} = K^{-1} W \Lambda\, W^T K^{-1T} \tag{B-6}$$

and the mean squared value of the midcourse maneuver is

$$u^2 = \overline{u_1^2 + u_2^2} = \overline{u_1^2} + \overline{u_2^2} \tag{B-7}$$

i.e. the sum of the two diagonal terms of the matrix on the right side of equation B-7. In order to cope with 99 per cent of all cases the spacecraft should have a maneuver capability between $2.1u$ and $2.5u$ depending on the dispersion ellipse of u_1 and u_2.

APPENDIX C

OUTLINE OF THE METHOD OF COMPUTING THE ORBIT

It is assumed that the actual trajectory is very close to a pre-computed reference trajectory, and consequently, that linear perturbation theory is applicable for deducing first approximations. Let $\delta\psi$ be an $(M \times 1)$ matrix denoting all M measured coordinate perturbations; M is a large number since it includes many measurements of angles and range-rate (and possibly range) from several tracking sites. The trajectory can be completely specified by perturbations in the six injection coordinates

$$\delta V = \begin{bmatrix} \delta V_1 \\ \delta V_2 \\ \delta V_3 \\ \delta V_4 \\ \delta V_5 \\ \delta V_6 \end{bmatrix} \tag{C-1}$$

Furthermore, from linear perturbation theory,

$$\delta\psi = U\, \delta V \tag{C-2}$$

where U is an $(M \times 6)$ matrix computed on the reference trajectory. Now $\delta\psi$ represents the coordinates that would be measured but for noise, actually the coordinates $\delta\xi$ are observed where

$$\delta\xi = \delta\psi + N \tag{C-3}$$

where

$$N = \begin{bmatrix} N_1 \\ N_2 \\ \cdot \\ \cdot \\ \cdot \\ N_M \end{bmatrix} \tag{C-4}$$

repres ents the noise components on all kinds of measurements at all times from all sites.

It is assumed that these noise components are associated with a multivariate Gaussian distribution, with the probability density function

$$P(N) = \frac{1}{\sqrt{(2\pi)^M |K|}} \exp(-\tfrac{1}{2} N^T K^{-1} N) \tag{C-4}$$

where K is the noise moment matrix of all M measurement-errors. By substituting equation C-3 in C-4

$$P(N) = \frac{1}{\sqrt{(2\pi)^M |K|}} \exp\{-\tfrac{1}{2}(\delta\xi - \delta\psi)^T K^{-1} (\delta\xi - \delta\psi)\} \tag{C-5}$$

Computation of δV by the method of maximum likelihood (ref.(1) consists of maximizing $P(N)$ or, what amounts to the same thing, minimizing

$$(\delta\xi - \delta\psi)^T K^{-1} (\delta\xi - \delta\psi) \tag{C-6}$$

The solution for the (6×1) matrix δV, which minimizes expression C-6, can be shown to be

$$\delta V = J^{-1} U^T K^{-1} \delta\xi \tag{C-7}$$

where

$$J = U^T K^{-1} U \tag{C-8}$$

However K is an $(M \times M)$ matrix, where M may be as high as 1000; it would be impracticable to invert numerically such a high order matrix. Fortunately, it would usually be possible to assume that all the measurement errors are uncorrelated, in which case K becomes simply a diagonal matrix of the variances of the noise on the different kinds of measurements at different times. (In practice, if certain errors are self-correlated, it is sufficient to modify the variances in the diagonal K matrix according to the correlation interval.) The method of maximum likelihood then becomes the met hod of weighted least-squares and only the (6×6) J matrix has to be inverted. For very accurate determination of orbits it would not usually be sufficient to rely on linear perturbation theory. Instead an iterativ e procedure would be employed where each iteration is calculated as above.

An important corrolary to equation C-7 is deduced as follows: equation C-3 is substituted in C-7

$$\delta V = J^{-1}U^{T}K^{-1}\delta\psi + J^{-1}U^{T}K^{-1}N \tag{C-9}$$

i.e. δV has been expressed as the sum of the true solution plus the uncertainty due to noise components. Let the latter by δV_u, then

$$\delta V_u = J^{-1}U^{T}K^{-1}N \tag{C-10}$$

and

$$\delta V_u \delta V_u^T = J^{-1}U^{T}K^{-1}NN^{T}K^{-1T}UJ^{-1T} \tag{C-11}$$

Taking the ensemble average, noting that $K = \overline{NN^T}$ and using equation C-8,

$$\delta V_u \delta V_u^T = J^{-1} \tag{C-12}$$

which is the noise-moment matrix of the uncertainties in determining the six initial coordinates.